

Abstract

COVID-19 has impacted public and economic health worldwide. To bolster the economy and maintain human life, economic and epidemiological research is vital. Nations have implemented lockdowns intent on slowing the spread of the virus. This research examines how lockdown parameter adjustments can help control a nation's fatalities. The study incorporated an SIRD disease model that is simulated over a 365 day period. The goal of the research is to take the SIRD model and incorporate further parameters to simulate a lockdown. Being able to observe the outcome of the model simulation may provide insight to the importance of lockdown intensity for the future. I hope to use this model to create a minimization function that analyzes dynamics that best produce minimal loss of GDP as well as low loss of life in a lockdown.

Methods

This research utilizes a compartmental SIR type model. This model is comprised of three groups; susceptible, infected, and recovered. To follow similar studies, a deceased group was added making it an SIRD model. A diagram of this can be seen in FIGURE 1.

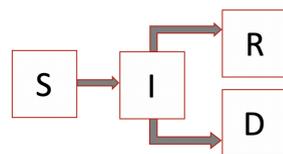


Figure 1. SIRD Diagram

N represents the populations and $N(0) = 1$ is the population at time $t = 0$ meaning 100% of the population proportion. Individuals move from susceptible $S(t)$ to infected $I(t)$ and then to either recovered $R(t)$ or deceased $D(t)$. N is fixed because $S' + I' + R' + D' = 0$. The dynamics of the SIRD model are represented as a system of differential equations which can be seen below with parameters.

Equations & Parameter Breakdown

$$S(t) + I(t) + R(t) + D(t) = N(t), \text{ when } t \geq 0.$$

$$\begin{aligned} S'(t) &= -\beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)) && \text{Susceptible population} \\ I'(t) &= \beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)) - \gamma I(t) && \text{Infected population} \\ R'(t) &= \gamma I(t) - \phi(I(t))I(t) && \text{Recovered population} \\ -N'(t) &= D'(t) = \phi(I(t))I(t) && \text{Deceased population.} \end{aligned}$$

β is the infection rate set to 0.2.

γ is the recovery rate representing a 18 day recovery.

$L(t)$ is the population percentage in lockdown between $[0, 70\%]$.

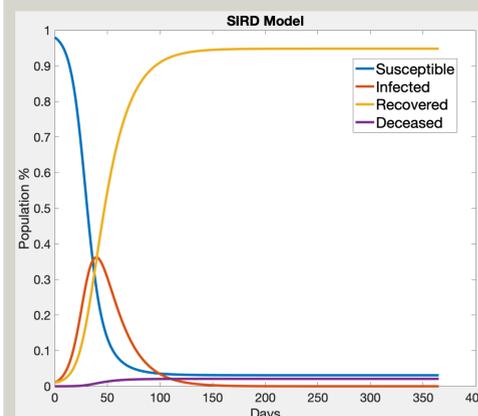
θ is the obedience level of the lockdown which has been set to 50%.

$\phi(I)$ is the death rate function $\phi(I(t)) = \varphi + kI(t)$ dependent on the infection number.

- $\varphi = 0.01\gamma$ representing the base fatality rate per active case per day.
- $k = 0.05\gamma$ represents the increase in death rates as hospitals get more crowded.

Model Assessment

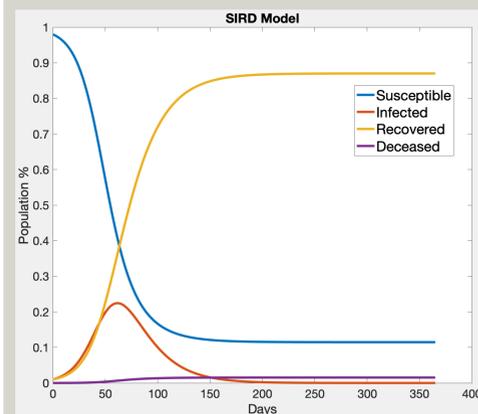
The SIRD model was numerically solved using mathematical software MATLAB. Lockdown parameters were adjusted to analyze how different levels of a lockdown can potentially produce different outcomes in a nation's peak infection rate as well as total fatalities over a 365 day period. Graphs of these models can be seen in the figures below.



| L Parameter | Max Infected | Deceased |
|-------------|--------------|----------|
| 0% | 36.2% | 2.09% |

When there is no lockdown we have a peak infection rate of about 36.2% after 39 days. Under this lockdown level 2.09% of the population would be deceased after 365 days. We can see in FIGURE 2 that the infection rate resembles a bell curve and when it comes to disease modeling the goal is to flatten that curve.

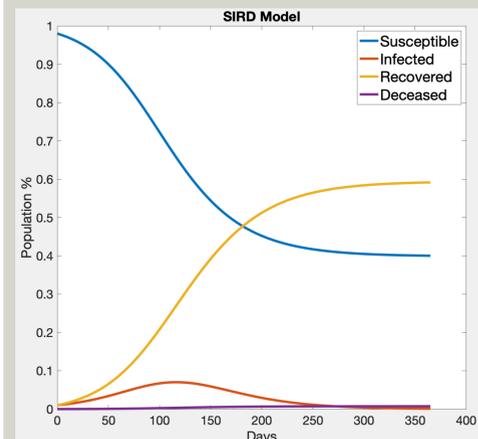
Figure 2. No Lockdown Level



| L Parameter | Max Infected | Deceased |
|-------------|--------------|----------|
| 35% | 22.43% | 1.53% |

When the lockdown parameter is set to 35%, we have a peak infection rate of about 22.43% after 62 days. Under this lockdown level 1.53% of the population would be deceased after 365 days. We can see in FIGURE 3 that the curve is starting to flatten out with the level of lockdown rising.

Figure 3. Minimal Lockdown Level



| L Parameter | Max Infected | Deceased |
|-------------|--------------|----------|
| 70% | 7.01% | 0.73% |

We will consider a nation is under full lockdown when the lockdown parameter is set to 70% to account for about 30% of the population being essential workers. At this max lockdown, there is a peak infection rate of 7.01% after 116 days. Under this lockdown level 0.73% of the population would be deceased after 365 days. We can see in FIGURE 4 that when the lockdown is at 70% we would have significantly flattened the curve.

Figure 4. Full Lockdown Level

Discussion

Working with the SIRD model has gone smoothly. I have graphed the SIRD model and replicated similar results as Alvarez et al. (2020). This model shows how a pandemic lockdown can develop over a 365 day period. In order to map a lockdown, assumptions must be made. We are assuming that the obedience level of the lockdown is constant at 50% which is unrealistic. We are also assuming that the death rate function will be constant through the pandemic. Lastly, when this research began vaccines were not accessible to the population. Because of this a vaccine parameter was not included. In the future I would like to add further parameters in this model.

The Next Step

The next phase of the research is to produce a minimization function to help decide how to balance deaths brought on by COVID-19 with economic losses. With the population dynamics from the SIRD model, we can create an optimization problem with the objective of minimizing current costs brought on by COVID-19 per person during a lockdown. An example of such a minimization problem is given:

$$\min_{L \in [0, L]} \int_0^{\infty} e^{-rt} \left(\omega L(t) [S(t) + I(t)] + I(t) \phi(I(t)) \left[\frac{\omega}{r} \right] \right) dt.$$

In this minimization function r is representing the annual discount rate and ω is representing the unit output per person.

Conclusion

As we can see from the data in this report, the level of compliance from a given population is vital in order to keep deceased levels as low as possible during a pandemic. As we saw from the various lockdown levels, the higher the lockdown parameter the longer it will take to move the population from the susceptible group to the recovered group. But this higher lockdown level will result in a lower percentage of deceased which is priority. Research such as this is vital to better educate the public that the more compliant they are the shorter the duration of a lockdown will likely be while also helping to keep deceased levels low. I plan to conduct further research incorporating the minimization function. I am currently researching how higher obedience levels in a given population can produce shorter lockdown periods which will also result in less economic loss.

Acknowledgments

This undergraduate research project was funded by the Ronald E. McNair Post-Baccalaureate Achievement Program through a grant from the U.S. Department of Education.



References

- [1] Fernando E Alvarez, David Argente, and Francesco Lippi. A simple planning problem for covid-19 lockdown. Technical report, National Bureau of Economic Research, 2020.
- [2] Erhan Bayraktar, Asaf Cohen, and April Nellis. A macroeconomic sir model for covid-19. *Mathematics*, 9(16):1901, 2021.
- [3] Lindsay Joan Martin. *Methods for Solving Hamilton-Jacobi-Bellman Equations*. PhD thesis, 2019.