Quantitative Properties of 1-Bridge Braids

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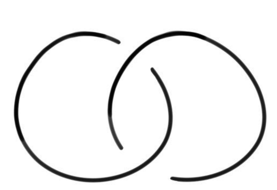
Knots and Braids

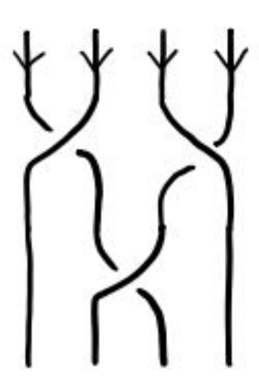
Definitions:

- A **knot** is a potentially knotted circle in 3-dimensional space.

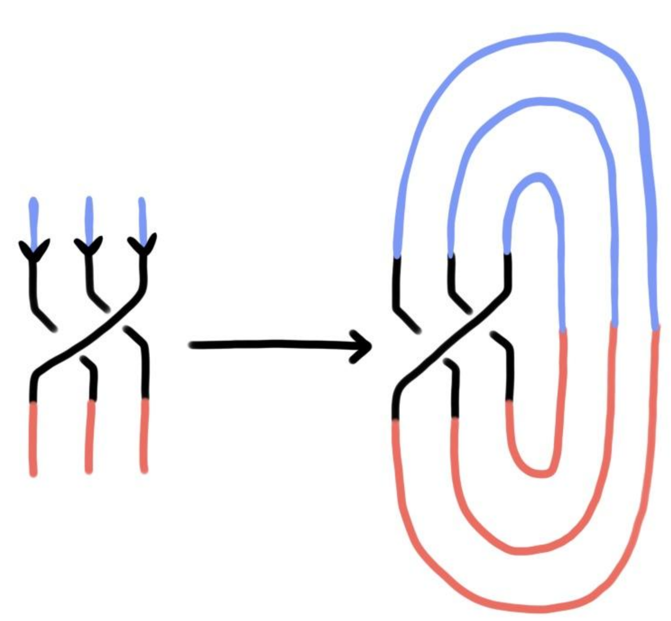
- A **link** is a collection of knots.



- An ***n*-stranded braid** is a collection of *n* parallel strands with crossings between adjacent stands.



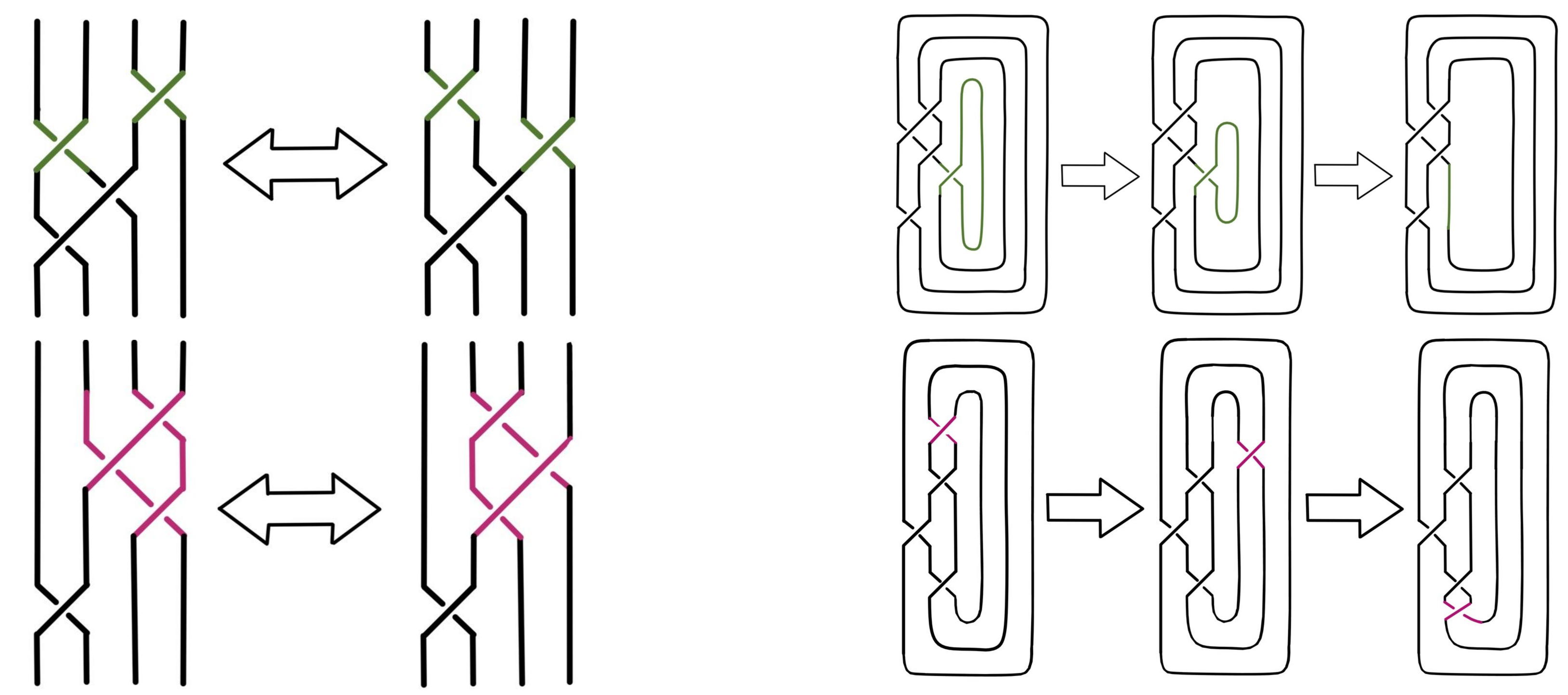
Braid Closures



Fact (Alexander's Theorem):
Every knot is realized as the closure of some braid.

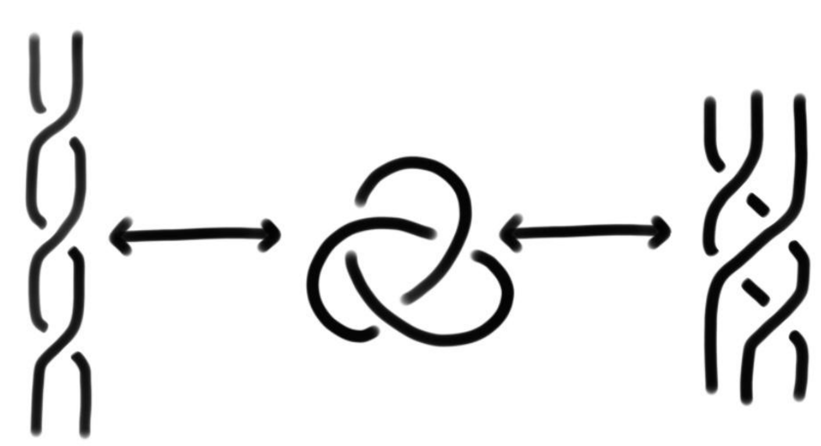
Markov's Theorem

Different braids can have the same closure. **Markov's theorem** tells us how these braids are related.



The Braid Index

The **braid index of a knot *K*,** denoted *i(K)*, is the fewest number of strands required to present *K* as the closure of some braid.

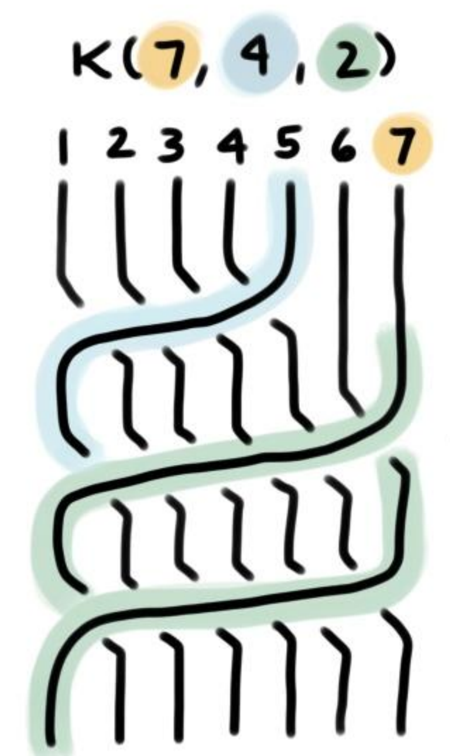


In general, it is difficult to determine the braid index of an arbitrary knot.

1-Bridge Braids

A **1-bridge braid, *K(w,b,t)*,** is formed by taking the closure of a braid defined via three parameters:

- *w* = # of strands
- *b* = bridge number
- *t* = twist number



Studied in [1], these knots have interesting Dehn surgery properties.

Our Project

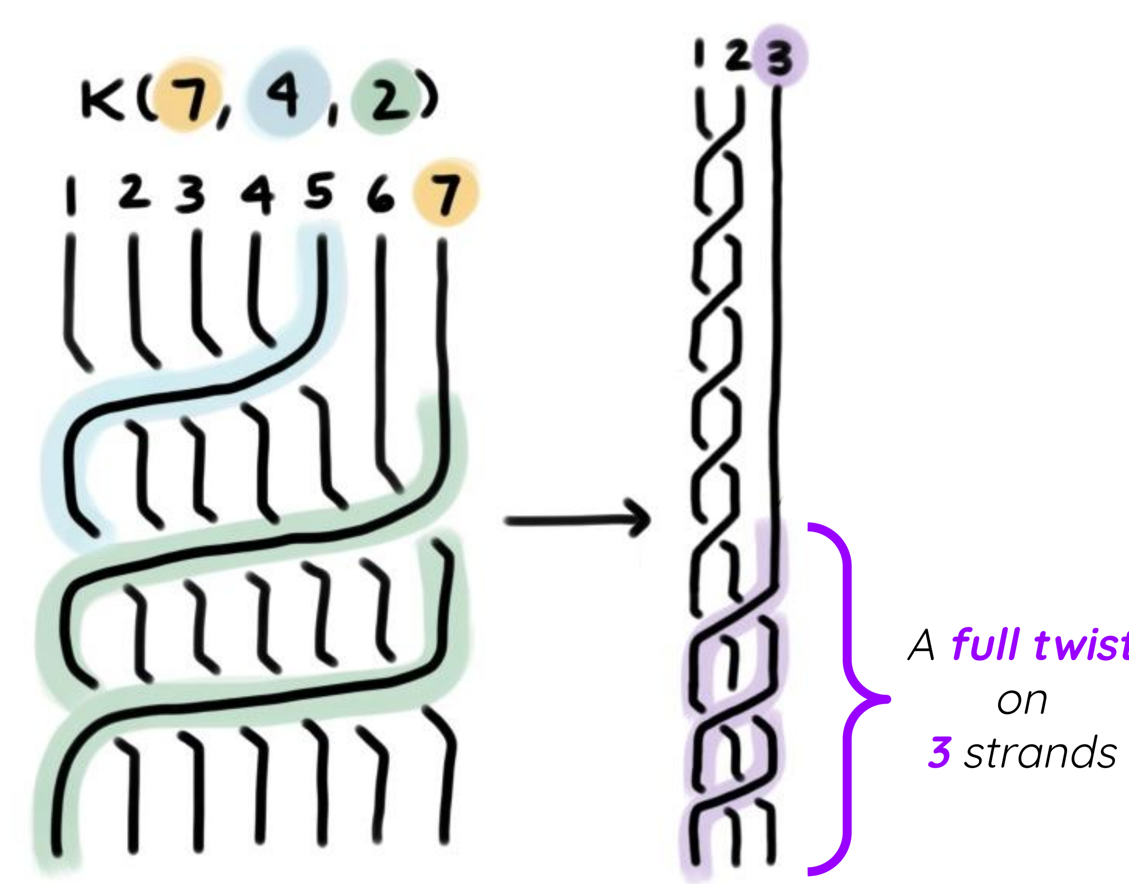
Question: What is *i(K(w,b,t))*?

Theorem (G-K-L-N-T-W): *The braid index of a 1-bridge braid K(w,b,t) is determined by its parameters:*

$$i(K(w,b,t)) = \begin{cases} w & t \geq w \\ t & w > t > b \\ t+1 & b \geq t \end{cases}$$

Proof Sketch:

1. Use Markov moves to rewrite braid.
2. Use the **Morton; Franks-Williams Theorem**: *if ω is a positive braid word on n strands, and β≈ω*(**full twist**), i(β) = n.*



References

1. David Gabai, *1-bridge braids in solid tori*, Topology Appl.37(1990), no. 3, 221–235
2. H. R. Morton, *Seifert circles and knot polynomials*, Math. Proc. Cambridge Philos. Soc.99(1986), no. 1, 107–109.MR 809504
3. John Franks and R. F. Williams, *Braids and the Jones polynomial*, Trans. Amer. Math. Soc.303(1987), no. 1,97–108. MR 896009

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