Surviving Ragnarök:
Modeling Humanity's Chance of Survival After a Major Disaster Event

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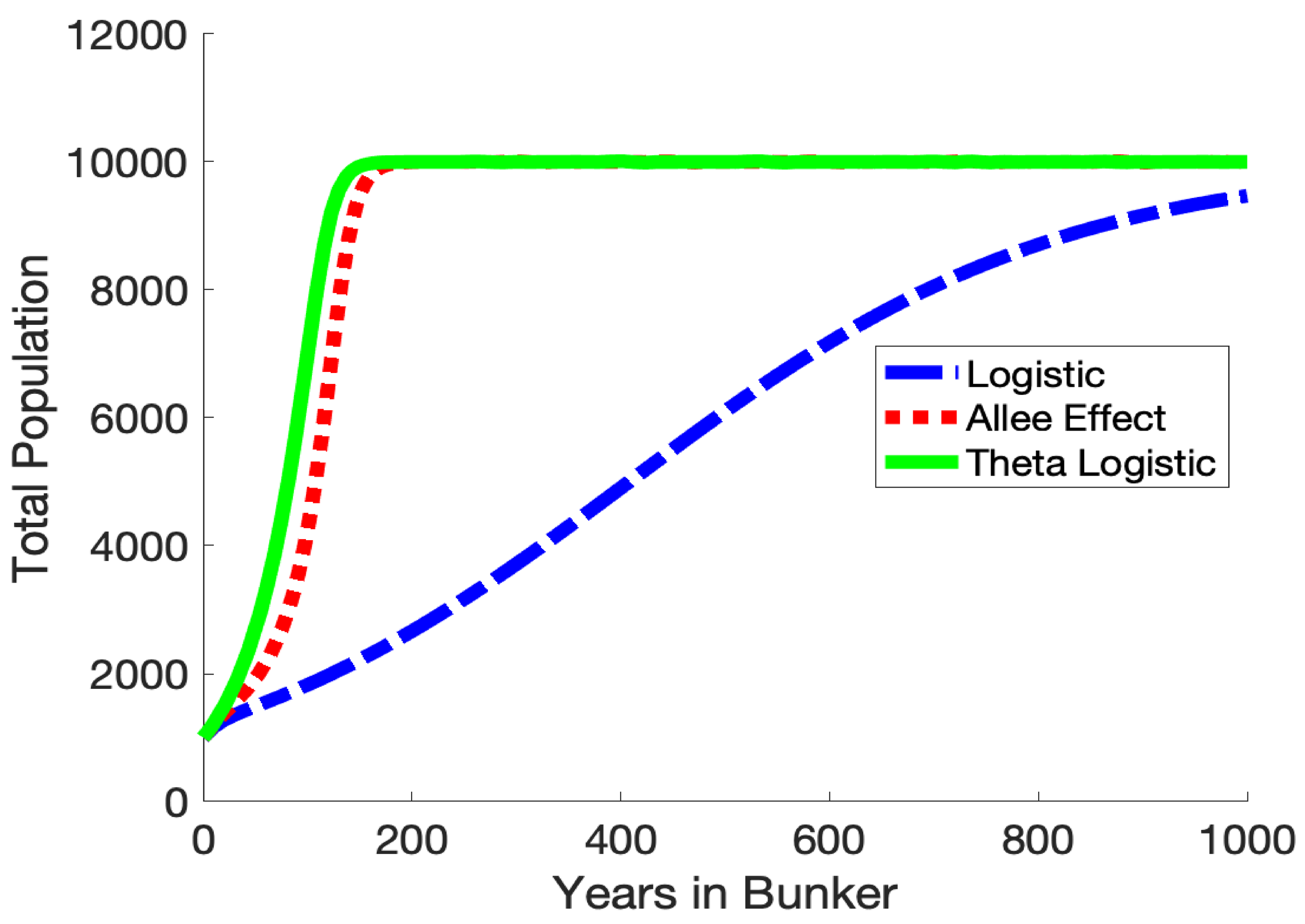
Abstract

Our research models the growth of a population from an initial number of survivors following a catastrophic event by using Markov population chains and differential equations while the population is living within a bunker. We then make use of stochastic models to study the dynamics of the population once it leaves the bunker after a set period of time. With these models, we establish a viable range for the initial population that ensures a steady growth rate.



Differential Models

- Where $K = 10{,}000$
- $\alpha = 500$
- $\theta = 5$
- $r = 24.268e^{-0.1t} - 9.649e^{-0.3t} + 4.8692$

Regular Logistic Model

$$\frac{dP}{dt} = \frac{r}{1000}P\left(1 - \frac{P}{K}\right)$$

Theta Logistic Model

$$\frac{dP}{dt} = \frac{r}{1000}P\left[1 - \left(\frac{P}{K}\right)^{\theta}\right]$$

Allee Effect Model

$$\frac{dP}{dt} = \frac{r}{1000}P\left(1 - \frac{P}{K}\right)\left(\frac{P}{\alpha} - 1\right)$$

Equations and Models

In order to tackle this problem, we took two approaches:

- First, we gathered population data from the US Census, CDC, and the NSA

When then used this data to construct different types of models:

- Differential Equation Models
  - Logistic Equation
  - Theta Logistic Equation
  - Allee-Effect Logistic Equation
- Matrix Population Models:
  - Leslie Matrix
  - Logistic Matrix Model (Jensen)

After comparing all our models, we decided to focus on the Allee Effect and logistic matrix models.

The problem:

Events such as natural disasters, disease epidemics, and wars heighten our collective awareness of the fragility of human life on Earth. Therefore, it comes as no surprise that researchers have sought to create mathematical models to predict the behavior of population growth following these events.

- We began our research by considering the aftermath of the Yellowstone eruption.

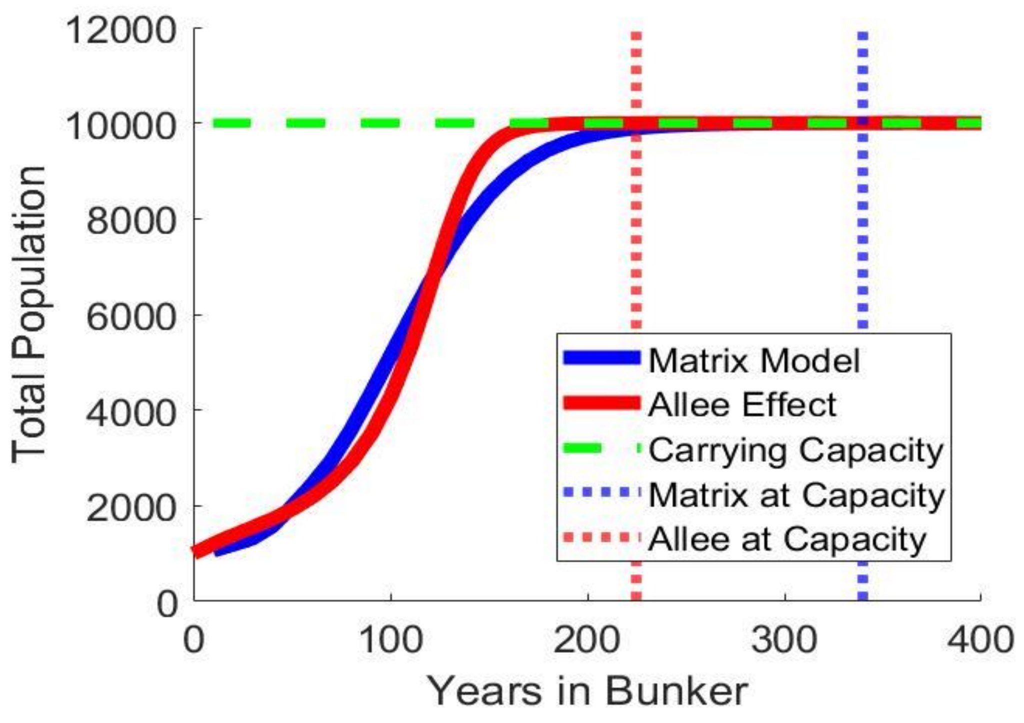
We decided to approach this problem in a two-step process:

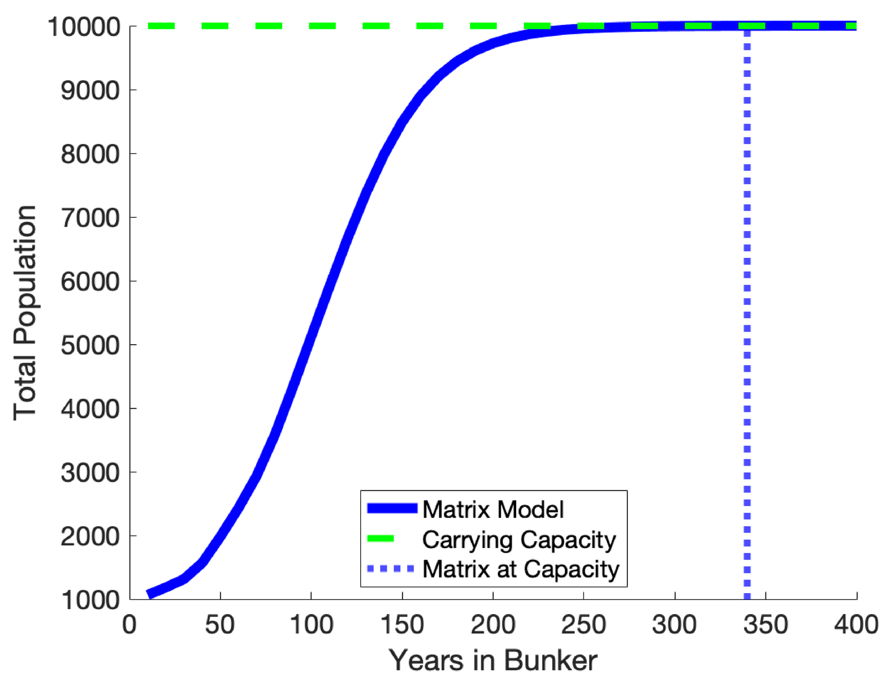
1. Inside the bunker:
   - The model created for this scenario serves as reference for the model for outside the bunker.
   - We also can make assumptions about the qualities and previsions in the bunker (such as carrying capacity, medicine, etc.).
2. Outside the bunker:
   - We incorporate both demographic and environmental stochasticity in order to model the environment outside of the bunker.
   - We also make use variance analysis to determine the probability of survival of population given an initial number of people.

Sources:

Social Security Administration. "Period Life Table 2017" https://www.ssa.gov/oact/STATS/table4c6.html

Center for Disease Control and Prevention. "Births: Final Data for 2019" https://www.cdc.gov/nchs/data/nvsr/nvsr70/nvsr70-02-508.pdf





Logistic Matrix Model proposed by Jensen

$$N(t+1) = N(t) + \left(1 - \frac{N}{K}\right)[\mathrm{L} - I]N(t)$$

Here, K is still the carrying capacity, $N(t)$ is the vector representing the population at time t, N is the total population, and I is the identity matrix.

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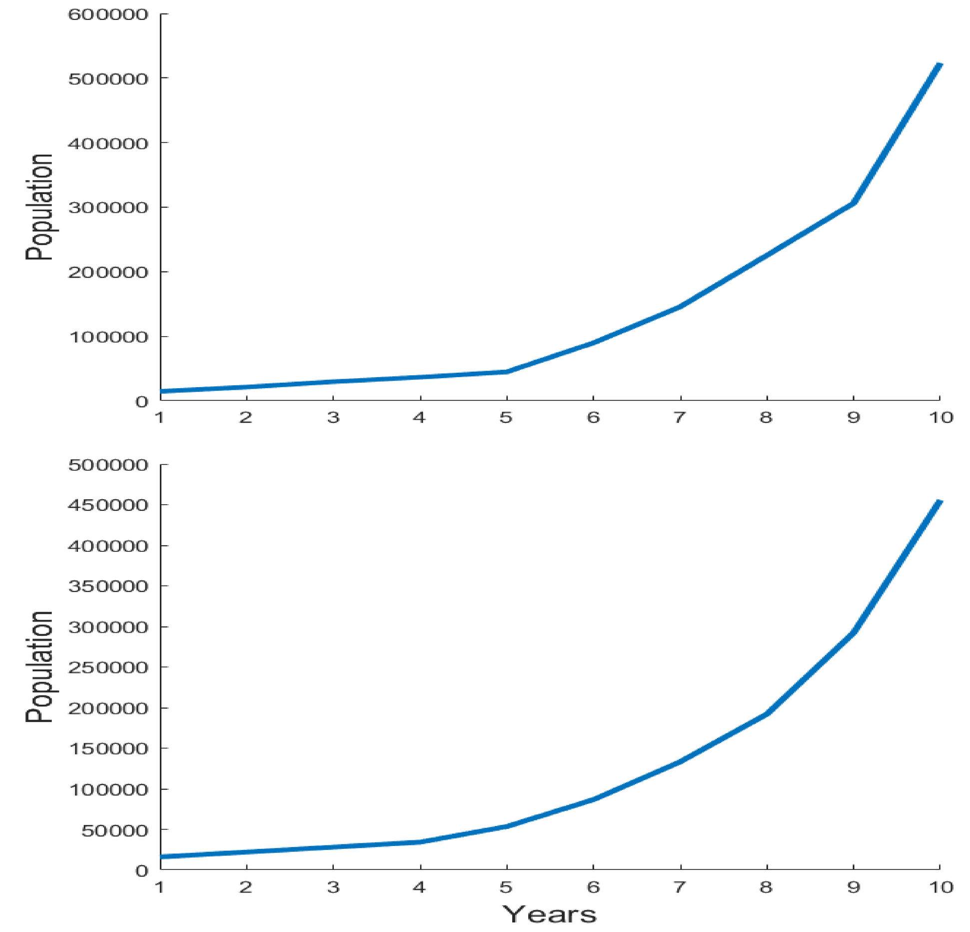
Ongoing Work:

The next step in our research is to leave the bunker. In order to simulate this, we make use of stochastic models.

- Initial population of 10000
- Fertility rates are calculated from a Poisson distribution, with mean 2.21.
- Survival rates are calculated from a Uniform Distribution with varying parameters by age.



The first graph depicts a single run of a ten-year span. The second graph shows the average population of 1000 runs over a ten-year span.

References:

P.H. Leslie. "On the Use of Matrices in Certain Population Dynamics" Biometrika, 33(3), 183–212, Nov 1945. https://doi.org/10.1093/biomet/33.3.183

H. Caswell. "Matrix Population Models" Encyclopedia of Environmetrics (eds A.H. El-Shaarawi, W.W. Piegorsch and C. Dean), 2006, https://doi.org/10.1002/9780470057339.vam006m

A.L. Jensen. "Simple density-dependent matrix model for population projection" Ecological Modeling, 77(1), 43-48, 1995. https://doi.org/10.1016/0304-3800(93)E0081-D