

Do Voting Paradoxes Occur in the Real World? The Case of AP College Football Polls

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Description of Data

- Data Source: AP College Football Poll Rankings, scraped from CollegePollTracker.com.
- 7 Seasons analyzed (2014-2020), 16-17 weekly polls per season, X polls in total

AP College Poll Explained

- 60-65 Pollsters, representing sports media outlets.
- Each pollster submits a ranked list of Top 25 teams each week during the college football season.
- Ballots are aggregated using the **Borda Count** method, which assigns 25 points for a first place vote, 24 points for a second place vote, and so on.
- Total number of points for each team determines the official AP Top 25 Ranking.

Voting Terminology

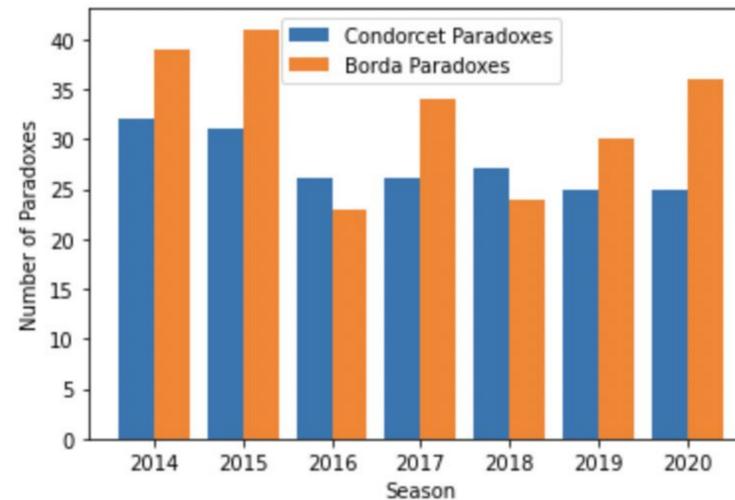
- **Plurality Winner:** The team which receives the most first place votes.
- **Borda Count Winner:** The team which receives the most Borda Count points.
- **Condorcet Winner:** The candidate which wins in head-to-head competition against every other candidate.
- **Condorcet Paradox:** The plurality winner is not the condorcet winner.
- **Borda Paradox:** The borda count winner is not the condorcet winner.
- **Cyclic Majority Paradox:** A triple of candidates (A,B,C), such that a majority of voters rank A above B, B above C, and C above A.

Methodology

- For each week in each of the 7 seasons analyzed, we considered all triples of teams among the AP Top 15 (resp. Top 25) for the occurrences of borda and condorcet (resp. cyclic majority) paradoxes.
- Regarding each such triple as a separate election, with votes tallied using the plurality, borda, and condorcet methods, we counted cases in which these methods led to different election winners or a cyclic majority.
- We disregarded triples which involved ties between 2 teams.

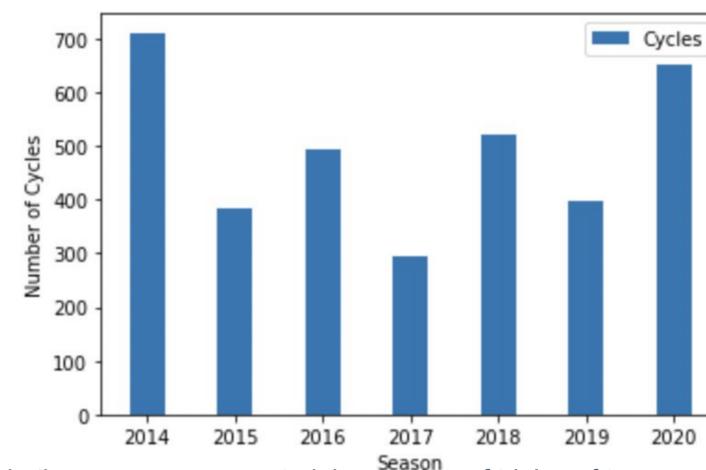
Results

Condorcet and Borda Paradoxes Among Top 15 Teams by Season



- For each week in the season, and each triple of teams among the top 15 teams in that week's AP Poll Ranking, we determined the plurality, Borda Count, and condorcet winners within this triple.
- There were a total of $C(15,3) * 17 = 7,735$ such triples of teams per season.
- The chart shows the total number of instances, among those 7,735 triples, in which the plurality winner is not the condorcet winner (**Condorcet paradox**), and the total number of instances in which the Borda Count winner is not the condorcet winner (**Borda paradox**).

Cyclic Majority Paradoxes Among Top 25 Teams by Season



- For each week in the season, we counted the number of triples of teams among the Top 25 teams in that week's AP Poll Ranking which constituted a **Cyclic Majority Paradox**.
- There were a total of $C(25,3) * 17 = 39,100$ such triples of teams per season.
- The chart shows the total number of instances, among those 39,100 triples, of the **Cyclic Majority Paradox** in each season.

Summary of Findings

- **Condorcet and Borda Paradoxes:** There were a total of 54,145 triples of teams among the AP Top 15 in our data set. Out of these 54,145 triples:
 - 192 (or 0.35%) resulted in a Condorcet paradox
 - 227 (or 0.42%) resulted in a Borda paradox
 - 86 (or 0.16%) resulted in both a Condorcet and Borda paradox.
- **Cyclic Majority Paradoxes:** There were a total of 273,700 triples of teams among the AP Top 25. Out of these triples, 3,455 (or 1.262%) were cycles. Limitations:
 - The majority of cycles found involved teams appearing near the bottom of the AP Top 25 and ranked by only a small number of pollsters. In fact, 3112 (or 90%) of the cycles found involved a team that was ranked by only 1 pollster.
 - 71 (or 2.05%) of the cycles found involved only teams that were ranked by at least 50% of pollsters.
 - 45 (or 1.30%) of the cycles found involved only teams that were ranked by all pollsters.

References

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