



# Automorphisms of the Fine Curve Graph

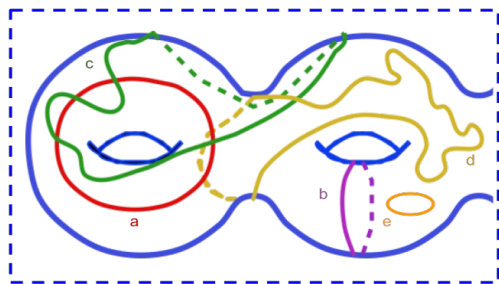
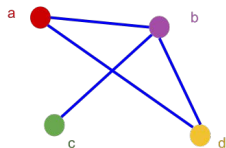


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## Fine Curve Graph $FC(S)$

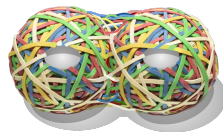
**Vertices:** essential simple closed curves in  $S$

**Edges:** Disjointness



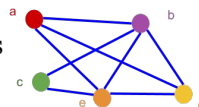
## Main Theorem

The natural map  $\text{Homeo}(S) \rightarrow \text{Aut } FC(S)$  is an isomorphism.



## Extended Fine Curve Graph $EFC(S)$

**Vertices:** simple closed curves (including inessential curves)

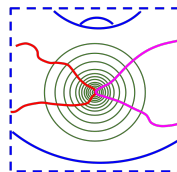


## Theorem (Farb-Margalit)

The natural map  $\text{Homeo}(S) \rightarrow \text{Aut } EFC(S)$  is an isomorphism.

## Subgraph of $EFC(S) \leftrightarrow$ Point in $S$

$(c_i) \rightarrow c \Leftrightarrow \forall a, b$  intersecting infinitely many  $c_i$ ;  $a$  intersects  $b$

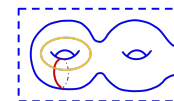


## Proof Approach: $EFC(S)$ to $FC(S)$

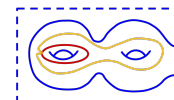
We want a map:  $\text{Aut } FC(S) \rightarrow \text{Aut } EFC(S) \stackrel{(F-M)}{=} \text{Homeo}(S)$

## Characterizing Curves

The **hull** of a set of curves: union of the curves and all the disks they bound



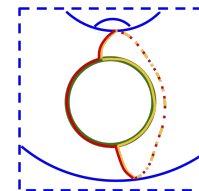
OR



$\Leftrightarrow$  The hull of two curves contains no other curve.

## Curve Pairs

Use essential curves characterizing inessential ones. More complex to characterize.



## Acknowledgements

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