Simple stage-structured population models

An ODE approach based on reproductive potential

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Introduction

Classical ODE Models

- Composed of distinct stages
- Reproductive variability must feature Exponential or Erlang distributed dwell-times.

New ODE Models

- Composed of distinct stages
- Reproductive variability can feature many distributed dwell-times, e.g. Pearson, Exponential, Gamma, Weibull, etc.
- Models feature mean residual waiting-times that account for the variability in reproduction

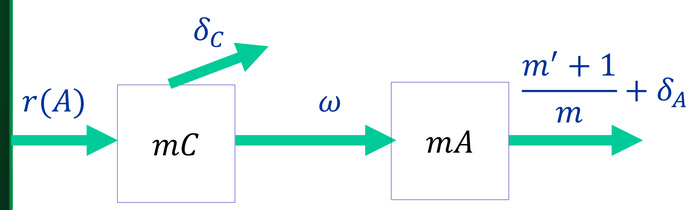

Figure 1. Compartmental diagram. The flow of the number of reproductive-days from Child to Adult compartments.

Methods

We developed a generalized compartmental model (Figure 1) of the Juvenile (C) and Adult (A) fish, Orange Roughy (Figure 2),

$$m\frac{dC}{dt} + \frac{dm}{dt}C = r(A)Am - (\omega + \delta_C)Cm,$$

$$m\frac{dA}{dt} + \frac{dm}{dt}A = \omega Cm - \left(\frac{m'+1}{m} + \delta_A\right)Am,$$

$$A = a_1 + a_2 m + a_3 m^2 + a_4 m^3.$$

Parameter details are available in Table 1.

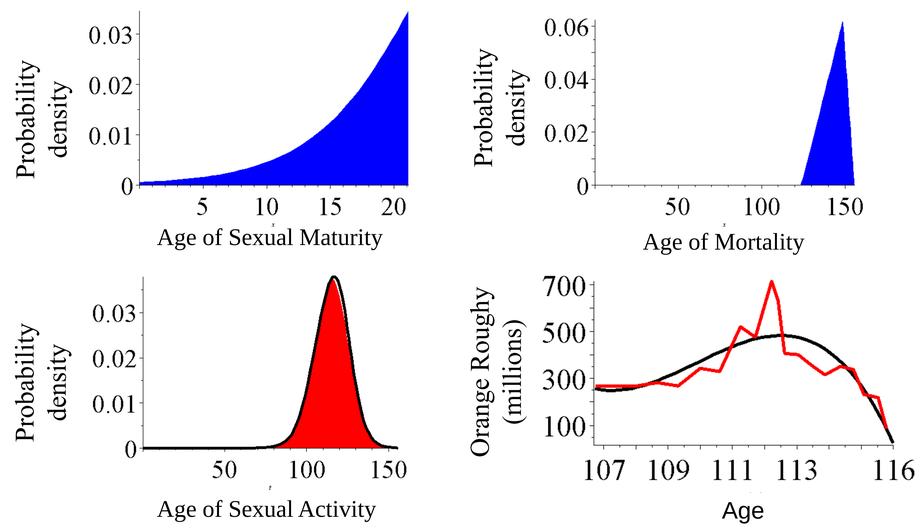

Figure 2. Probability density functions and population fit
Testing our model on the Orange Roughy species of fish, we created pdfs of the age of reaching sexual maturity, of the age of mortality, and age of sexual activity. The final graph is a fitting of the new model against Orange Roughy catch data.

Results

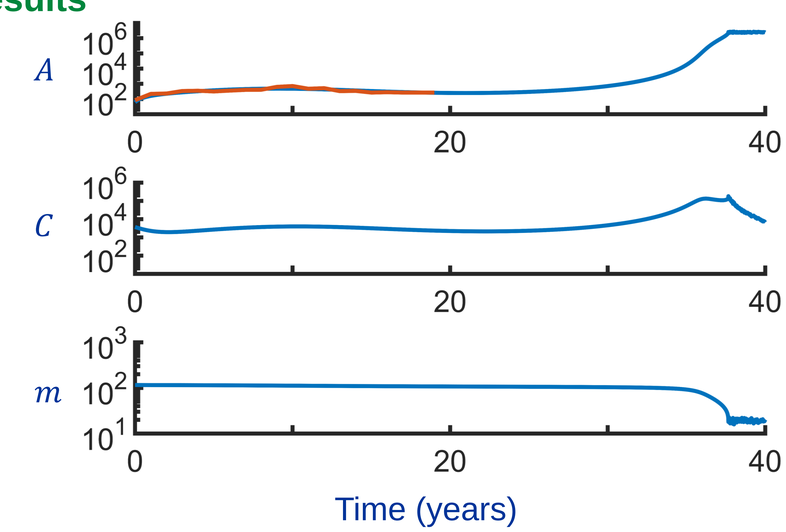

Figure 3. Population Fits. This graph displays the population size for the Adult Orange Roughy (red), and the population fits for Adult and Juvenile as well as the mean residual waiting time (m).

Parameter	Symbol	Base value
Per capita birth rate	$r(A) = be^{-\frac{A}{K}}$	
Fecundity	b	6564 year^{-1}
	K	60887
Avg. duration to sexual maturity	$1/\omega$	30.1 years
Avg. duration an individual can reproduce	$\frac{m'+1}{m}$	Figure 2d
Adult predation rate	δ_A	0.56 year^{-1}
Child predation rate	δ_C	0.01 year^{-1}
Parameters from the fit of $\alpha_1 + \alpha_2 m + \alpha_3 m^2 + \alpha_4 m^3 = A$		
Constant term	α_1	4.38×10^6
Linear coefficient	α_2	-1.2×10^5
Quadratic coefficient	α_3	1088.6
Cubic coefficient	α_4	-3.3

Table 1. Parameter names, symbols, and base values.

Discussion

Stock levels of the Orange Roughy appear stable, at least in the near future.

The Orange Roughy's duration of reproduction shows signs of little to no decline.

Future directions:

- Beverton-Holt and logistic type reproduction rates
- Applications to invasive species
  - Lionfish
  - Dipareses root weevil

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